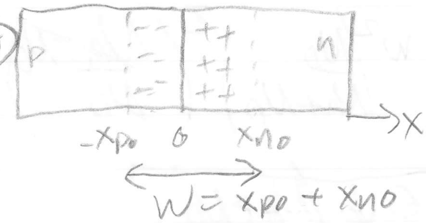
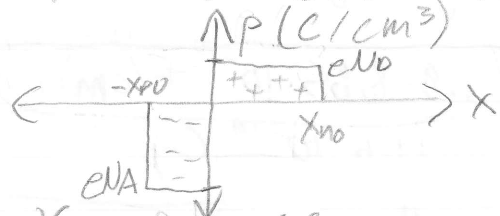
ECE 162B
HW #6 SOL'N

1(A)



$N_A = 2\cdot 10^{15}\ cm^{-3} \cong N_A^-$ } HEAVY
$N_D = 1\cdot 10^{15}\ cm^{-3} \cong N_D^+$ } DOPING
$N_A, N_D >> n_i$

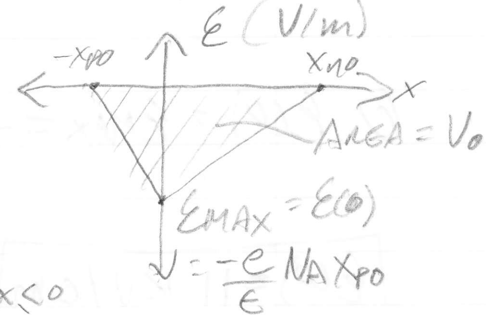
DEPLETION APPROXIMATION AND CHARGE NEUTRALITY REQUIRE:



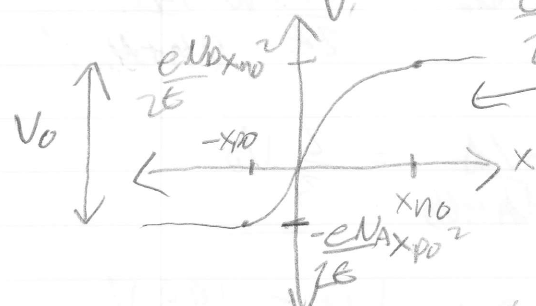
$$\underline{N_A \cdot x_{po} = N_D \cdot x_{no}}$$
$$\Rightarrow x_{no} = 2x_{po}$$

POISSON'S EQUATION: $\dfrac{\partial \mathcal{E}}{\partial x} = \dfrac{\rho}{\epsilon} = \begin{cases} -\dfrac{e}{\epsilon} N_A & \text{FOR } -x_{po} < x < 0 \\ \dfrac{e}{\epsilon} N_D & \text{FOR } 0 < x < x_{no} \end{cases}$

$$\Rightarrow \mathcal{E} = \begin{cases} 0 & x < -x_{po},\ x > x_{no} \\ -\dfrac{e}{\epsilon} N_A (x + x_{po}) & -x_{po} < x < 0 \\ \dfrac{e}{\epsilon} N_D (x - x_{no}) & 0 < x < x_{no} \end{cases}$$



$$\mathcal{E} = -\frac{dV}{dx} \Rightarrow V = \begin{cases} -\dfrac{e}{2\epsilon} N_A x_{po}^2 & x \le -x_{po} \\ \dfrac{e}{\epsilon} N_A \left(\tfrac{1}{2}x^2 + x \cdot x_{po}\right) & -x_{po} < x < 0 \\ -\dfrac{e}{\epsilon} N_D \left(\tfrac{1}{2}x^2 - x \cdot x_{no}\right) & 0 < x \le x_{no} \\ \dfrac{e N_D x_{no}^2}{2\epsilon} & x \ge x_{no} \end{cases}$$



NOTE: $-e \cdot V(x)$ GIVES BAND DIAGRAM

$$V_0 = \frac{e}{2\epsilon}\left(N_D x_{no}^2 + N_A x_{po}^2\right)$$

ALSO $\dfrac{n_n}{n_p} = e^{\left(\frac{eV_0}{kT}\right)}$ & $n_n \simeq N_D$ $\quad n_p \simeq \dfrac{n_i^2}{N_A}$ $\quad n_i = 1.5\cdot 10^{10}$ cm

$$\Rightarrow \frac{N_A N_D}{n_i^2} = e^{\frac{eV_0}{kT}} \Rightarrow V_0 = \frac{kT}{e} \ln\left(\frac{N_A N_D}{n_i^2}\right) = (.0259\ V) \ln\left(\frac{2\cdot 10^{30}}{2.25\cdot 10^{20}}\right)$$

(A) $\boxed{V_0 = 0.593\ V}$

$$N_A x_{p0} = N_D x_{n0} \qquad W = x_{n0} + x_{p0} = x_{n0}\left(1 + \frac{N_D}{N_A}\right)$$

$$W = x_{p0}\left(1 + \frac{N_A}{N_D}\right)$$

$$W = x_{p0}\left(\frac{N_A + N_D}{N_D}\right)$$

$$V_0 = \frac{e}{2\epsilon}\left(N_D\left(\frac{N_A}{N_D}x_{p0}\right)^2 + N_A x_{p0}^2\right)$$

$$= \frac{e}{2\epsilon} N_A x_{p0}^2\left(1 + \frac{N_A}{N_D}\right) = \frac{e N_A}{2\epsilon} W^2 \left(\frac{N_D}{N_A + N_D}\right)^2 \left(\cancel{\frac{N_A + N_D}{N_D}}\right)$$

$$= \frac{e W^2}{2\epsilon}\left(\frac{N_A N_D}{N_A + N_D}\right) \Rightarrow W = \sqrt{\frac{2\epsilon V_0}{e}\left(\frac{N_A + N_D}{N_A N_D}\right)}$$

$$W = \sqrt{\frac{2 \cdot (11.8 \cdot 8.85 \cdot 10^{-14}\ F/cm)(0.593\ V)}{(1.6 \cdot 10^{-19}\ C)}\left(\frac{3 \cdot 10^{15}\ cm^{-3}}{2 \cdot 10^{30}\ cm^{-6}}\right)}$$

(B) $W = 1.08 \cdot 10^{-4}\ cm = \boxed{1.08\ \mu m = W} \Rightarrow x_{n0} = \frac{2}{3}W = .718\ \mu m$

$x_{p0} = \frac{1}{3}W = .359\ \mu m$

(C) $\mathcal{E}(0) = \mathcal{E}_{MAX} = -\frac{e}{\epsilon} N_A x_{p0} = \frac{-e}{\epsilon} N_D x_{n0} = \frac{-1.6 \cdot 10^{-19}\ C \cdot (2 \cdot 10^{15}\ cm^{-3})(3.59 \cdot 10^{-5}\ cm)}{11.8 \cdot 8.85 \cdot 10^{-14}\ F/cm}$

$\boxed{\mathcal{E}(0) = 11\ kV/cm}$

(D) AT THE JUNCTION, CONSIDER $\frac{e N_D}{2\epsilon} x_{n0}^2 = \frac{e N_D W^2}{2\epsilon} \frac{N_A^2}{(N_A + N_D)^2}$

$$= \frac{\cancel{e} N_D N_A^2}{\cancel{2\epsilon}(N_A + N_D)^2} \cdot \frac{2\epsilon V_0}{e}\left(\cancel{\frac{N_A + N_D}{N_A N_D}}\right) = V_0 \frac{N_A}{N_A + N_D} = \frac{2}{3} V_0$$

SIMILARLY $\frac{e N_A}{2\epsilon} x_{p0}^2 = \frac{1}{3} V_0$, SO $V(0)$ IS $\frac{1}{3} V_0 = .198\ V$

GREATER THAN $V(x < -x_{p0})$, V IN THE P-BULK

(E) SEE EARLIER SKETCHES

(a) $N_D - N_A = Gx, \quad -W/2 < x < W/2$

$x_{po} = x_{no} = W/2$

AT ANY POINT IN THE JUNCTION, x, THE CHARGE DENSITY, $\rho(x) = e(N_D - N_A) = eGx$

$$\Rightarrow \frac{\partial \mathcal{E}}{\partial x} = \frac{\rho(x)}{\epsilon} = \frac{eGx}{\epsilon}$$ BY POISSON'S EQUATION

INTEGRATING $\Rightarrow \mathcal{E}(x) = \frac{eGx^2}{2\epsilon} + C$ APPLY B.C.'s $\mathcal{E}(\pm \frac{W}{2}) = 0$

$$\frac{eG}{8\epsilon} W^2 + C = 0 \Rightarrow C = -\frac{eGW^2}{8\epsilon}$$

$$\Rightarrow \boxed{\mathcal{E}(x) = \frac{eG}{2\epsilon}\left[x^2 - \left(\frac{W}{2}\right)^2\right]}$$

(B) $\mathcal{E}(x) = -\frac{dV}{dx} \Rightarrow V_0 - V = \int_{-W/2}^{W/2} -\frac{eG}{2\epsilon}\left[x^2 - \left(\frac{W}{2}\right)^2\right] dx$

$$V_0 - V = -\frac{eG}{2\epsilon}\left[\frac{x^3}{3} - \left(\frac{W}{2}\right)^2 x\right]_{-W/2}^{W/2} = -\frac{eG}{2\epsilon}\left[\frac{W^3}{12} - \frac{W^3}{4}\right] = \boxed{\frac{eGW^3}{12\epsilon} = V_0}$$

$$\Rightarrow \boxed{W = \left(\frac{12\epsilon(V_0 - V)}{eG}\right)^{1/3}}$$

(C) $C_j = \frac{\epsilon A}{W} = \epsilon A \left(\frac{eG}{12\epsilon(V_0 - V)}\right)^{1/3} = \boxed{A\left(\frac{eG\epsilon^2}{12(V_0 - V)}\right)^{1/3}} = C_j$